Synthesis and Experimental Accuracy Assessment of Kalman Filter Algorithm for UWB ToA Local Positioning System

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***Abstract* — This paper is devoted to synthesis and investigation of Kalman filter algorithm for ultrawideband (UWB) Time of Arrival (ToA) system positioning accuracy. The study consists of synthesis of positioning algorithm for UWB ToA local positioning system, experimental verification of proposed algorithm feasibility, assessment of statistical characteristics of coordiates estimates of developed filter in field experiment. Synthesized algorithm for processing UWB ToA measurements confirmed its feasibility in field experiment. Obtained standard deviation of user's coordinates estimates for proposed ToA Kalman filter algorithm amounted to 0.0115 m and 0.0161 m for coordinates X and Y, respectively. Found distance root mean square of synthesized ToA filter estimates is 0.0198 m which is 3.86 times less than DRMS for solutions from LSM.**

Keywords—UWB, Kalman Filter, ToA local positioning system.

I. Introduction

Nowadays the relevance of tracking the movements of workers and positioning of various mobile platforms, such as, for example, automatically guided vehicles, is increasing. Global navigation satellite systems (GNSS) [1] are well-established standard for positioning, navigation and timing support. However, due to the fact that most part of production processes take place indoors, where GNSS signals are not available, there is a need for alternative systems that can provide positioning in such conditions. These systems are usually called indoor positioning systems (IPS).

Currently, all known IPS can be divided by the type of technologies, on which they are built, into the following groups: optical systems [2], ultrasonic systems [3], inertial systems [4], odometric systems [5], magnetometric systems [6] and radio systems [7]. The advantages and disadvantages of each of the above systems are discussed in detail in [8] and are omitted from the narration in this article.

Radio systems seem to be the most suitable and equivalent in properties replacement for GNSS. Known IPS options based on the following radio standards are Wi-Fi, Bluetooth / Bluetooth low energy (BLE), ZigBee, Radiofrequency identification (RFID), NFER (Near-Field Electromagnetic Ranging), Ultra-WideBand (UWB). Of all the radio systems, it is worth highlighting the ultra-wideband radio systems, since they have the highest potential accuracy among all the above standards.

The UWB signal is a sequence of short pulses, due to which it is possible to effectively deal with the multipath problem that is urgent for the IPS. It consists in the fact that pulses corresponding to the direct signal path can be overlapped with pulses re-reflected from walls and various obstacles. In systems based on UWB signals, the probability of such an overlap is minimized (see Fig. 1) and their range resolution is sufficiently high, due to which their positioning error less than 10 cm and they are most attractive for a number of practical problems.

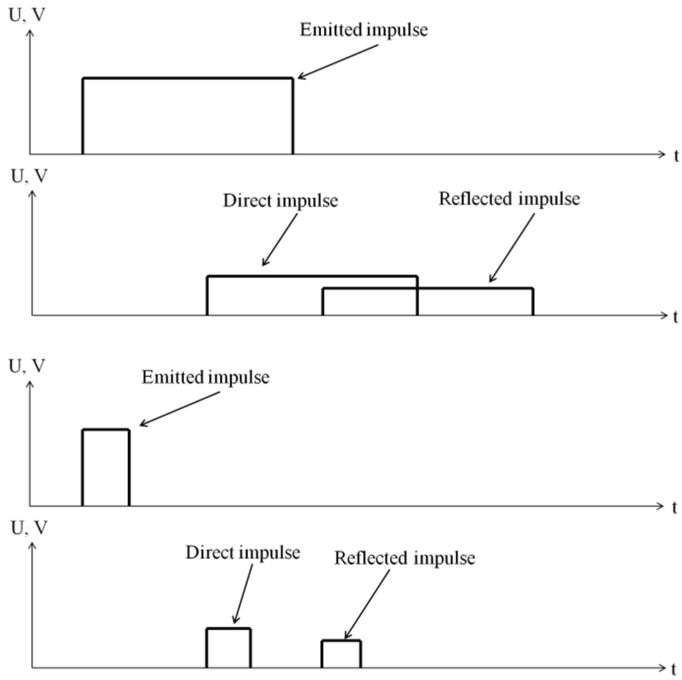

Fig.1. Advantage of ultrawideband signal over narrowband signal

Positioning in UWB-based IPS is implemented by intersection navigation methods. Intersection methods can be implemented in systems with different architectures, in which the measured parameters are: Time of Flight (ToF), Time of Arrival (ToA), Time Difference of Arrival (TDoA), Angle of Arrival (AoA), etc. All positional navigation methods are based on determining the unknown user location by one (or several) measured parameters, carried out by receiving a signal from radio navigation reference points (frequently called anchors) with previously known coordinates.

In this study we focused on radio system based on ToA measurements and the rest of the paper is devoted to them.

II. Problem Statement

Main purposes of this study is:

- to synthesize positioning algorithm for UWB ToA local positioning system;

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- experimentally verify proposed algorithm feasibility;
- to assess statistical characteristics of coordinates estimates of proposed filter in field experiment.

Problem statement for algorithm synthesis is the following.

Let assume that there is a user of ToA indoor positioning system which is needed to be positioned in two-dimensional local coordinate system. ToA indoor positioning system infrastructure consists of set of *N* static radio navigation points (anchors) with known (determined beforehand) coordinates (x_1, y_1), …, (x_N, y_N) and tag (tracked device which position corresponds to user position).

The working principle of ToA system under consideration consists in following. Tag transmits blink messages with predefined rate. All deployed anchors receive messages from tag and records time stamp corresponding to this event, each in its own time scale. Then, obtained measurements are corrected by estimated time shifts which are determined on calibration packets transmitted from master-anchor to the rest of anchors.

Thus, taking into account working principle of ToA system, its observation model is written as:

$$\mathbf{y}_k = \begin{bmatrix} c \cdot ToA_1 \\ \vdots \\ c \cdot ToA_N \end{bmatrix} + \mathbf{n}_k. \qquad (1)$$

where $ToA_1, \dots, ToA_N$ – are ToA measurements from 1…*N* anchor, *c* – speed of light and $\mathbf{n}_k$ is additive white Gaussian noise vector with zero mean and standard deviation σ_n.

III. ToA ALGORITHM

One of the most widely used, well-studied and well-established in practice approaches for processing measurements and obtaining user coordinates estimates in solution of navigation tasks is Kalman filtering [9].

This statistical approach allows to estimate state of dynamical system by noisy measurements. So before operating it implies developing observation model, state-transition model and setting covariance of observation and process noises.

The state of system in each moment of time can be described by set of parameters which is called state vector and is written as follows:

$$\mathbf{x}_k = \begin{bmatrix} x_k \\ V_{x_k} \\ y_k \\ V_{y_k} \\ T_{tr_k} \\ V_{\Delta_k} \end{bmatrix}, \qquad (2)$$

where x_k, y_k – coordinates of tracked object, V_{x_k}, V_{y_k} – components of velocity vector of tracked object, T_{tr_k} – mode of transmission of tracked object (it always increases as time-scale), V_{Δ_k} – rate of change of object mode of transmission T_{tr}.

Dynamic of state vector can be described as shown below:

$$\begin{cases} x_k = x_{k-1} + V_{x_{k-1}} T \\ V_{x_k} = V_{x_{k-1}} + \xi_{Vx_k} T \\ y_k = y_{k-1} + V_{y_{k-1}} T \\ V_{y_k} = V_{y_{k-1}} + \xi_{Vy_k} T \\ T_{tr_k} = T_{tr_{k-1}} + V_{\Delta_k} T \\ V_{\Delta_k} = V_{\Delta_{k-1}} + \xi_{\Delta_k} T \end{cases}, \qquad (3)$$

where $\xi_{Vx_k}, \xi_{Vy_k} \xi_{\Delta_k}$ – process noises for corresponding components of user's velocity vector and for rate of change of object mode of transmission. They are modelled as white Gaussian noises with following covariance $\sigma^2_{\xi_{V_x}}, \sigma^2_{\xi_{V_y}}, \sigma^2_{\xi_\Delta}$ respectively and *T* is system operating rate which in practice can be determined as difference between mean of two observation vector in two adjacent time steps.

Expression (2) can be found in vector-matrix form as:

$$\mathbf{x}_k = \mathbf{F}\mathbf{x}_{k-1} + \mathbf{G}\boldsymbol{\xi}_k \qquad (4)$$

where $\boldsymbol{\xi}_k$ – process noise vector, **G** – process noise transition matrix, **F** – state-transition matrix.

All the above-mentioned matrices can be found as following:

$$\mathbf{F} = \begin{bmatrix} 1 & T & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & T & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & T \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix},$$

$$\mathbf{G} = \begin{bmatrix} 0 & 0 & 0 \\ T & 0 & 0 \\ 0 & 0 & 0 \\ 0 & T & 0 \\ 0 & 0 & 0 \\ 0 & 0 & T \end{bmatrix}, \qquad (5)$$

$$\boldsymbol{\xi}_k = \begin{bmatrix} \xi_{Vx} \\ \xi_{Vy} \\ \xi_{\xi_\Delta} \end{bmatrix}.$$

At every *k*-th iteration Kalman filter provide extrapolated estimates of state vector $\tilde{\mathbf{x}}_k$ and covariance matrix $\widetilde{\mathbf{D}}_k$ on prediction step as follows:

$$\tilde{\mathbf{x}}_k = \mathbf{F}\hat{\mathbf{x}}_{k-1}, \\ \widetilde{\mathbf{D}}_k = \mathbf{F}\widehat{\mathbf{D}}_{k-1}\mathbf{F}^{\mathrm{T}} + \mathbf{G}\mathbf{D}_\xi\mathbf{G}^{\mathrm{T}}, \qquad (6)$$

where $\hat{\mathbf{x}}_{k-1}$ is state vector estimate of Kalman filter in previous time step, $\widehat{\mathbf{D}}_{k-1}$ is covariance matrix of estimated parameters in previous time step and $\mathbf{D}_\xi$ is process noise covariance matrix which can be written as shown below:

$$\mathbf{D}_\xi = \begin{bmatrix} {\sigma_{\xi_{V_x}}}^2 & 0 & 0 \\ 0 & {\sigma_{\xi_{V_y}}}^2 & 0 \\ 0 & 0 & {\sigma_{\xi_\Delta}}^2 \end{bmatrix}. \qquad (7)$$

Estimates of state vector $\hat{\mathbf{x}}_k$ and covariance matrix $\widehat{\mathbf{D}}_k$ can be calculated at every correction stage of algorithm:

$$\begin{aligned}\hat{\mathbf{x}}_k &= \tilde{\mathbf{x}}_k + \mathbf{K}_k(\mathbf{y}_k - \mathbf{S}_k),\\ \widehat{\mathbf{D}}_k &= \widetilde{\mathbf{D}}_k - \mathbf{K}_k\mathbf{H}_k\widetilde{\mathbf{D}}_k,\end{aligned} \tag{8}$$

where $\mathbf{H}_k$ – observation matrix, $\mathbf{K}_k$ – Kalman filter gain, $\mathbf{S}_k$ – functional dependence measurements on elements of state vector. The values above can be found as:

$$\mathbf{S}_k = \boldsymbol{f}(\tilde{\mathbf{x}}_k) =$$

$$= \begin{bmatrix} \tilde{T}_{tr\,k} + \sqrt{(\tilde{x}_k - x_1)^2 + (\tilde{y}_k - y_1)^2} \\ \vdots \\ \tilde{T}_{tr\,k} + \sqrt{(\tilde{x}_k - x_N)^2 + (\tilde{y}_k - y_N)^2} \end{bmatrix} \tag{9}$$

$$\mathbf{H}_k = \frac{\mathbf{dS_k}}{\mathbf{dx}} =$$

$$= \begin{bmatrix} \frac{(\tilde{x}_k - x_1)}{\sqrt{(\tilde{x}_k - x_1)^2 + (\tilde{y}_k - y_1)^2}} & \cdots & \frac{(\tilde{y}_k - y_1)}{\sqrt{(\tilde{x}_k - x_1)^2 + (\tilde{y}_k - y_1)^2}} \\ 0 & \cdots & 0 \\ \frac{(\tilde{x}_k - x_N)}{\sqrt{(\tilde{x}_k - x_N)^2 + (\tilde{y}_k - y_N)^2}} & \cdots & \frac{(\tilde{y}_k - y_N)}{\sqrt{(\tilde{x}_k - x_N)^2 + (\tilde{y}_k - y_1)^2}} \\ 0 & \cdots & 0 \\ 1 & \cdots & 1 \\ 0 & \cdots & 0 \end{bmatrix}^T \tag{10}$$

$$\mathbf{K}_k = \widetilde{\mathbf{D}}_k\mathbf{H}_k^T(\mathbf{H}_k\widetilde{\mathbf{D}}_k\mathbf{H}_k^T + \mathbf{D}_n)^{-1}, \tag{11}$$

$$\mathbf{D}_n = \begin{bmatrix} {\sigma_n}^2 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & {\sigma_n}^2 \end{bmatrix}, \tag{12}$$

where $\mathbf{D}_n$ – covariance matrix of observations with covariance ${\sigma_n}^2$ on main diagonal and the rest of the elements are zeros) and size of which is equal to number of measurements available on given time step.

IV. Experiment

To verify proposed algorithm feasibility, we held out field experiment. The conducted experiment consisted of two parts. In first part statistical characteristics of coordinates estimates in the output of proposed ToA filter under stationary conditions were obtained. In second part we investigated coordinates estimates of developed ToA filter under controlled dynamic conditions – UWB tag is moved along known path. Conditions of carried out experiment are presented below.

Four UWB anchors were placed on positions with following coordinates in local coordinate system: (0, 0, 1.5), (0, 2.91, 1.5), (3.97, 3.08, 1.5), (3.97, -0.46, 1.5).

UWB tag was fixed to rotational stand before experiment. In the first part of experiment we obtained and processed measurements from the tag remained in stationary position during test. In the second part of experiment the tag was moved manually on circular trajectory as shown on Figure 2 below.

UWB tag blink rate was set to 0.1 s. The standard deviation of measurement noise σ_n equaled to 0.5 ns. The values in process noise covariance matrix, form of which presented above in (7), were empirically chosen and matrix itself looks as shown below:

$$\mathbf{D}_\xi = \begin{bmatrix} 0.01 & 0 & 0 \\ 0 & 0.01 & 0 \\ 0 & 0 & 0.0005 \end{bmatrix}. \tag{7}$$

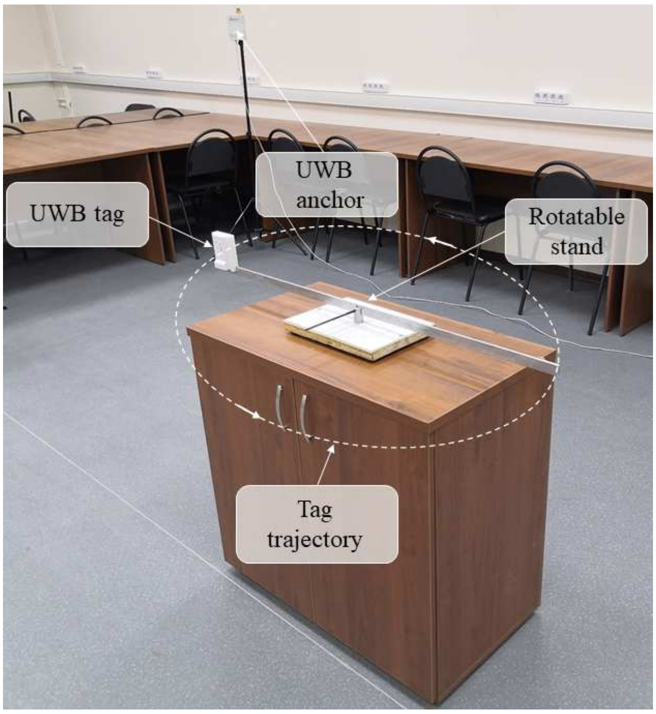

Figure 2. Experimental stand.

V. Experiment results

Results obtained in second part of conducted experiment is shown on Fig. 3. As described above in this part of tests we investigate behavior of coordinates estimates of proposed filter when the tag was moved along known (circular) path.

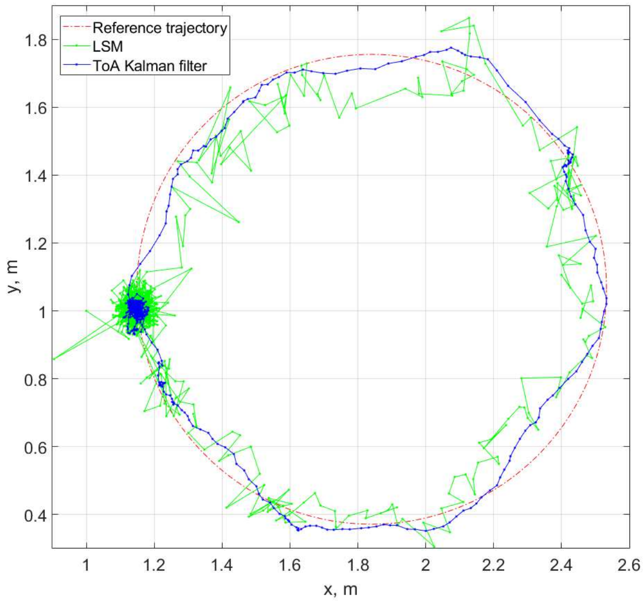

Fig. 3. Coordinates estimates of LSM and ToA Kalman filter obtained in experiment (movement on circular path).

In view of the fact there was not measuring equipment with help of which we could determine in what point of trajectory the tag is located in given moment of time, we can't correctly assessed and bring quantitative metrics of proposed filter under dynamic conditions, but we can affirm with complete confidence that shape of trajectory constructed on coordinates estimates (blue line) mimics the shape of referenced path (red line) and it is much smoother (and more akin to referenced trajectory) than trajectory obtained by solutions of LSM (green line).

Results obtained in first part of carried out experiment is presented on Fig. 4. In this part of tests we assessed statistical characteristics of coordinates estimates of proposed filter when the tag remained in stationary position.

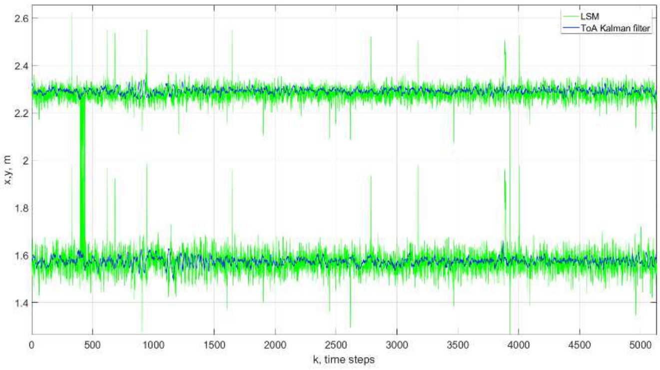

Fig. 4. Coordinates estimates of LSM and ToA Kalman filter obtained in experiment (stationary position).

The statistical characteristics (standard deviation and distance root mean square error) of estimates of LSM and proposed ToA Kalman filter algorithm obtained in experiment with tag in stationary position is summarized in Table 1.

TABLE I. STANDARD DEVIATION OF TAG COORDINATES ESTIMATES OF LSM AND PROPOSED TOA KALMAN FILTER ALGORITHM

		Standard deviation of coordinates estimates, m	***Distance Root Mean Square (DRMS), m***
LSM	σ_x	0.0483	0.0765
	σ_y	0.0594	
ToA Kalman filter	σ_x	0.0115	0.0198
	σ_y	0.0161	

VI. CONCLUSION

Synthesized algorithm for processing UWB ToA measurements confirmed its feasibility in field experiment. Obtained standard deviation of user's coordinates estimates for proposed ToA Kalman filter algorithm amounted to 0.0115 m and 0.0161 m for coordinates X and Y, respectively. Found distance root mean square of synthesized ToA filter estimates is 0.0198 m which is 3.86 times less than DRMS for solutions from LSM.

As the next steps in future work in this area, it is planned to implement NLOS detection and mitigation approach in existing algorithm and integrate UWB tag radio module with IMU to increase positioning accuracy by combined processing of inertial and radio measurements.

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